

Solution to Assignment 3

Supplementary Problems

1. Express the straight line $ax + by = 1$, $a, b > 0$, in polar coordinates. What is the range of θ ?

Solution. Let $c = \sqrt{a^2 + b^2}$. Equation is

$$1 = r(a \cos \theta + b \sin \theta) = rc \left(\frac{a}{c} \cos \theta + \frac{b}{c} \sin \theta \right) = rc \sin(\theta + \alpha),$$

where $\alpha \in (0, \pi/2)$ satisfies $\sin \alpha = a/c$. We have

$$r = \frac{1}{c \sin(\theta + \alpha)},$$

and r is positive only when $\theta + \alpha \in (0, \pi)$. Therefore, the straight line is given by

$$r = \frac{1}{c \sin(\theta + \alpha)}, \quad \theta \in (-\alpha, -\alpha + \pi).$$

2. Express the hyperbola $x^2 - y^2 = 1$ ($y \geq 0$) in polar coordinates.

Solution. From $1 = r^2(\cos^2 \theta - \sin^2 \theta) = r^2 \cos 2\theta$ we get

$$r = \frac{1}{\sqrt{\cos 2\theta}},$$

where $\theta \in (-\pi/4, \pi/4)$.